Hybrid Event Shaping to Stabilize Periodic Hybrid Orbits

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Motivation

Touchdown impacts that occur during legged locomotion cause instantaneous changes to system dynamics.

These hybrid events cause discrete, unbounded changes to trajectory tracking errors.

In this work we present a method to quantify the effect of hybrid events on orbital stability and control hybrid event parameters to stabilize periodic trajectories.

Saltation Matrix

The **saltation matrix**, Ξ, maps perturbations across hybrid events.

Ξ is a function of nominal event state, time, and higher order "**shape parameters**".

These shape parameters are independent of the nominal dynamics of the system, but do affect the perturbation dynamics (e.g. velocity of a massless leg at touchdown).

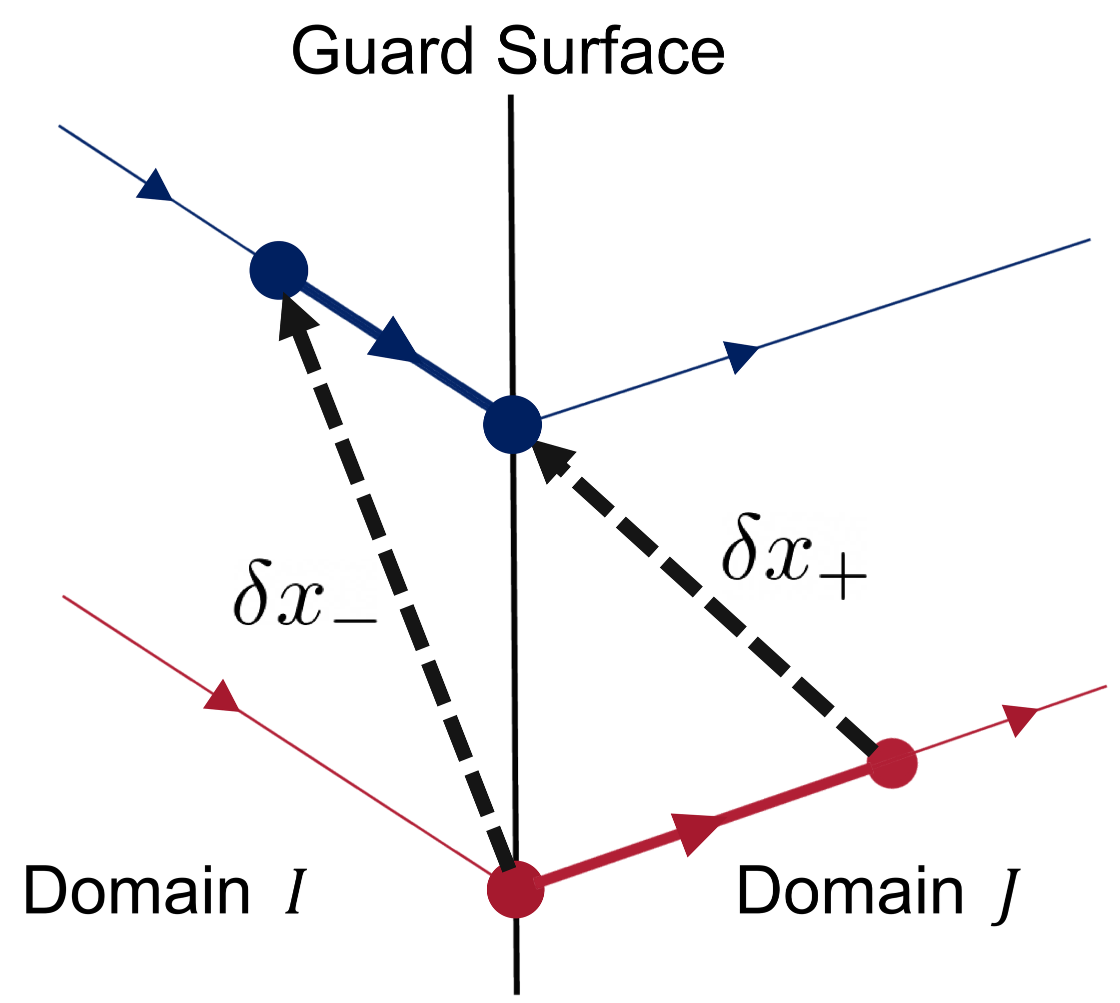

Fig. 1: A hybrid event discretely affects perturbation dynamics as the system passes across a guard surface. The saltation matrix, Ξ, maps the changes in these perturbations.

Hybrid Event Shaping

Shape parameters of the saltation matrix can be chosen freely to optimize the stability measure with no effect on the nominal trajectory.

This is **hybrid event shaping**: selecting favorable parameters of the hybrid event to induce stability in the overall orbit.

Hybrid event shaping is a generalizable, scalable method that can be applied to a broad range of hybrid systems.

For instance, we use hybrid event shaping to unite several previous works, like paddle juggling (Fig. 2) [1] and primitives for legged locomotion, shown in the following section.

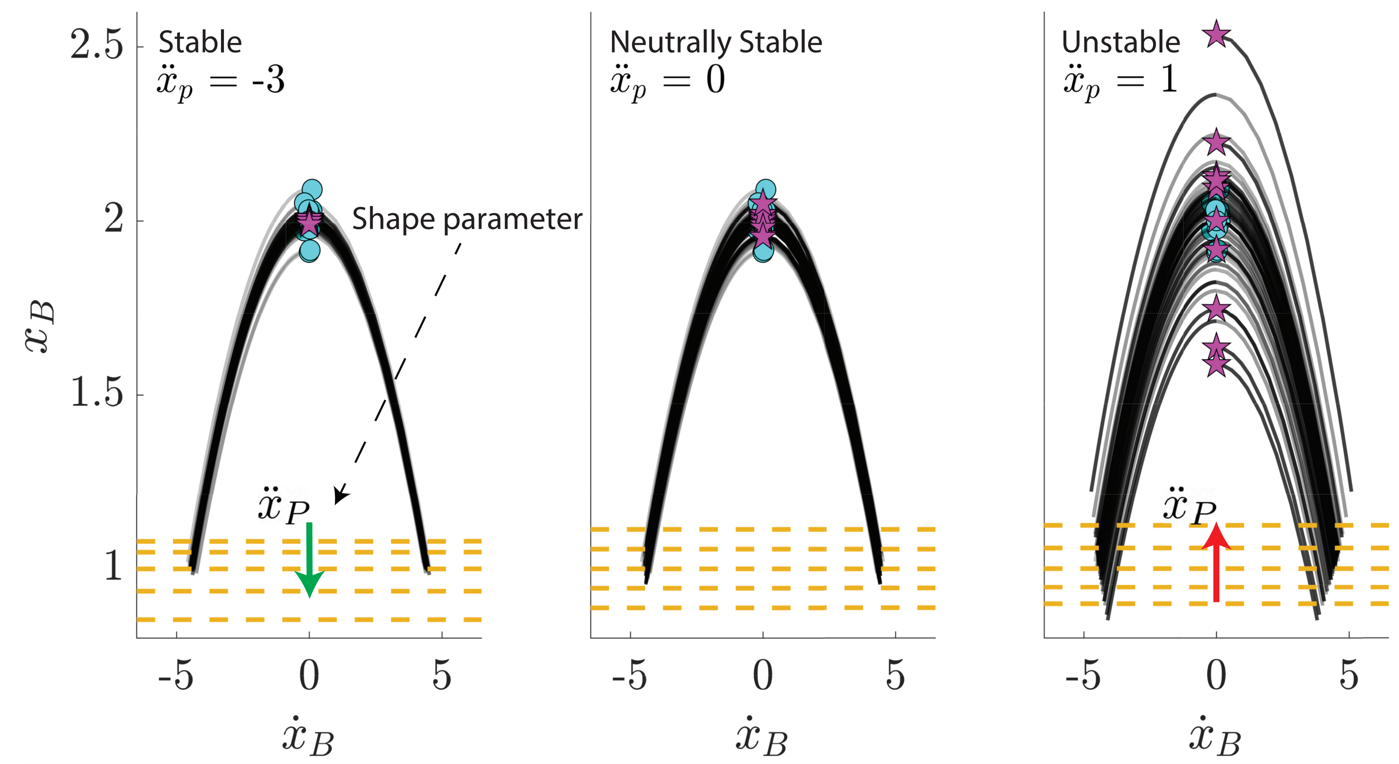

Fig. 2: For the paddle juggler system [1], the shape parameter that determines stability is the paddle acceleration, $\ddot{x}_P$. Hybrid event shaping solves for optimizal stabilizing values of this shape parameter.

Example and Results

Consider a 2D hopper with a point mass body and massless leg consisting of a spring, damper, and actuator in parallel.

Choosing the following leg angle controller introduces shape parameters in the touchdown saltation matrix:

$$\theta_{td} = \bar{\theta}_{td} + K(\dot{x} - \bar{\dot{x}}) + \omega(t - \bar{t}_{td})$$

Optimizing the stability measure over K and ω generates a stable solution (Fig, 3) that replicates the well-known Raibert heuristic [2] and swing leg retraction phenomenon [3].

Unlike the previous methods that were generated from observation and tuning, hybrid event shaping systematically gives exact stabilizing values for these shape parameters.

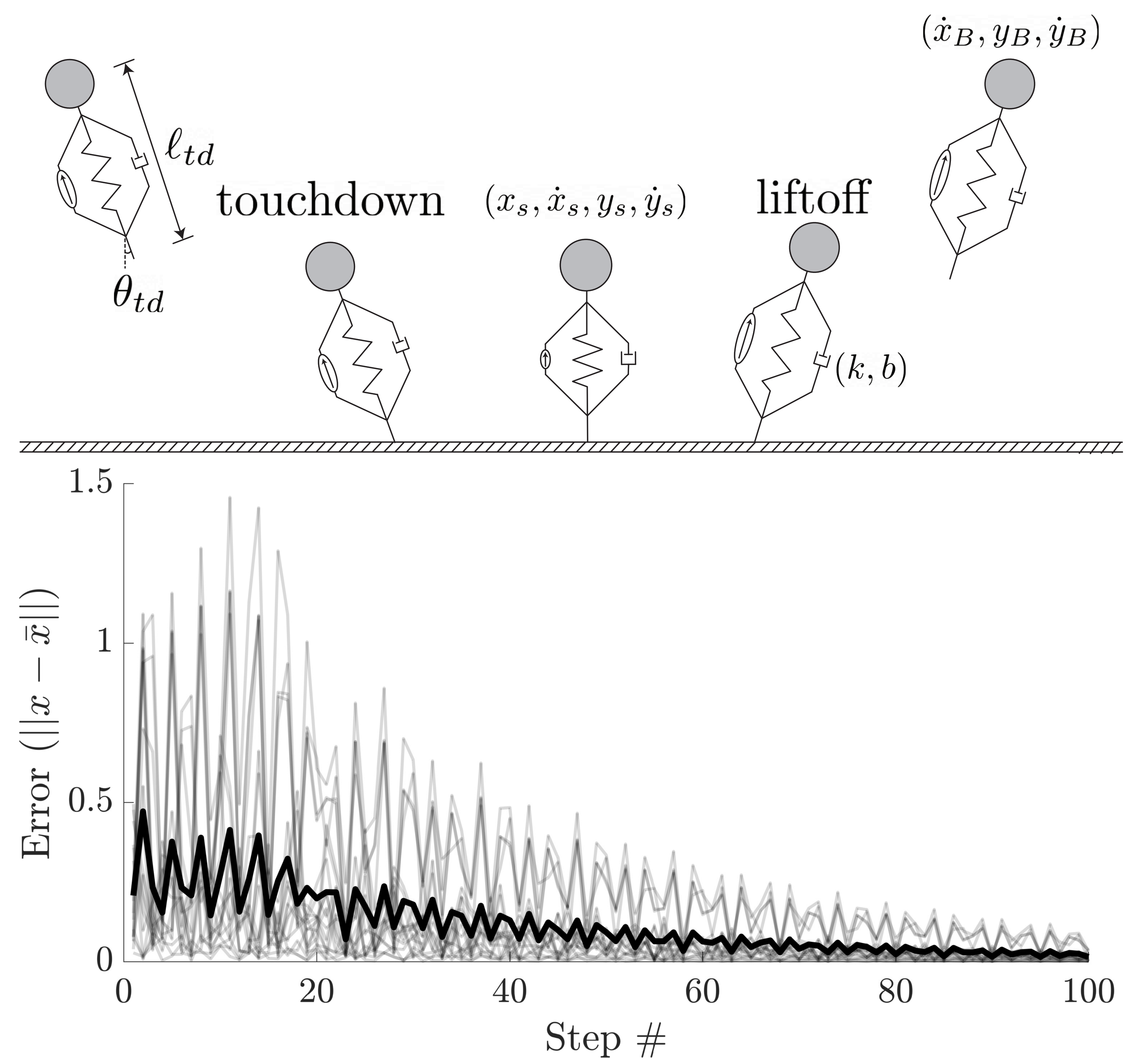

Fig. 3: Simulation of 50 trials (grey) shows hybrid event shaping stabilizes the SLIP hopping gait by independently re-discovering swing leg retraction and the Raibert heuristic. Bold black line indicates mean error.

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[2] M. Raibert, et. al., "Experiments in balance with a 3D one-legged hopping machine," The International Journal of Robotics Research 1984.
[3] Y. Blum, S. Lipfert, J. Rummel, and A. Seyfarth, "Swing leg control in human running," Bioinspiration & Biomimetics, vol. 5, June 2010.

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